Homework 2.3

2.3/1-19
2010-09-15

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41) $\lim_{x\to-\infty} f(x) = 7 \qquad \lim_{x\to-\infty} g(x) = -6$

e) $\lim_{x\to-\infty} \sqrt[3]{f(x)\,g(x)}$

$= \sqrt[3]{\lim_{x\to-\infty} f(x) \cdot \lim_{x\to-\infty} g(x)}$

$= \sqrt[3]{(7)(-6)}$

$= \sqrt[3]{-42}$

f) $\lim_{x\to-\infty} \frac{g(x)}{f(x)}$

$\frac{\lim_{x\to-\infty} g(x)}{\lim_{x\to-\infty} f(x)} = \frac{-6}{7}$

g) $\lim_{x\to-\infty} \left[f(x) + \frac{g(x)}{x}\right]$

$\lim_{x\to-\infty} f(x) + \lim_{x\to-\infty} \frac{g(x)}{x}$

$7 + \lim_{x\to-\infty} \frac{g(x)}{x}$

$= 7 + 0 = 7$

h) $\lim_{x\to-\infty} \frac{x\,f(x)}{(2x+3)\,g(x)}$ $\quad \frac{-\infty}{\infty}$ IDK

$= \lim_{x\to-\infty} \frac{x\,(f(x))}{x\left(2+\frac{3}{x}\right)g(x)}$

$= \lim_{x\to-\infty} \frac{f(x)}{\left(2+\frac{3}{x}\right)g(x)}$

$= \frac{7}{(2)(-6)} = -\frac{7}{12}$

15)

$$\lim_{x\to-\infty}\frac{x-2}{x^2+2x+1}$$

$$\lim_{x\to-\infty}\frac{x\left(1-\frac{2}{x}\right)}{x^2\left(1+\frac{2}{x}+\frac{1}{x^2}\right)}$$

$$\lim_{x\to-\infty}\frac{\left(1-\frac{2}{x}\right)}{x\left(1+\frac{2}{x}+\frac{1}{x^2}\right)}\qquad\begin{array}{l}\to 1\\ \to -\infty\end{array}$$

$$=0$$

$$\lim_{x\to-\infty}\frac{x^2\left(\frac{1}{x}-\frac{2}{x^2}\right)}{x^2\left(1+\frac{2}{x}+\frac{1}{x^2}\right)}$$

$$=\lim_{x\to-\infty}\frac{\left(\frac{1}{x}-\frac{2}{x^2}\right)}{1+\frac{2}{x}+\frac{1}{x^2}}=\frac{0}{1}$$

$$\lim_{x\to-\infty}\frac{x\left(1-\frac{2}{x}\right)}{x\left(x+2+\frac{1}{x}\right)}$$

$$=\lim_{x\to\infty}\frac{1-\frac{2}{x}}{x+2+\frac{1}{x}}$$

$$=0$$

17) $\lim\limits_{x \to +\infty} \sqrt[3]{\dfrac{2+3x-5x^2}{1+8x^2}}$

$= \lim\limits_{x \to +\infty} \sqrt[3]{\dfrac{x^2\left(-5+\frac{3}{x}+\frac{2}{x^2}\right)}{x^2\left(8+\frac{1}{x^2}\right)}}$

$= \lim\limits_{x \to +\infty} \sqrt[3]{\dfrac{\left(-5+\frac{3}{x}+\frac{2}{x^2}\right)}{\left(8+\frac{1}{x^2}\right)}}$

$= \sqrt[3]{\dfrac{-5}{8}} = \dfrac{\sqrt[3]{-5}}{\sqrt[3]{8}} = \dfrac{\sqrt[3]{5}}{2}$

$\sqrt[3]{\dfrac{x^2\left(-5+\frac{3}{x}+\frac{2}{x^2}\right)}{x^2\left(8+\frac{1}{x^2}\right)}}$

$= \sqrt[3]{\dfrac{\left(-5+\frac{3}{x}+\frac{2}{x^2}\right)}{\left(8+\frac{1}{x^2}\right)}}$

$\lim\limits_{x \to \infty} \sqrt[3]{\dfrac{-5+\frac{3}{x}+\frac{2}{x^2}}{8+\frac{1}{x^2}}}$

$= \sqrt[3]{\dfrac{-5}{8}}$

19)

$$\lim_{x\to-\infty} \frac{\sqrt{5x^2-2}}{x+3} \cdot \frac{\sqrt{5x^2-2}}{\sqrt{5x^2-2}}$$

$$\lim_{x\to-\infty} \frac{(5x^2-2)}{(x+3)\sqrt{5x^2-2}}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2\left(5-\frac{2}{x^2}\right)}}{x\left(1+\frac{3}{x}\right)}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{x^2}\sqrt{5-\frac{2}{x^2}}}{x\left(1+\frac{3}{x}\right)}$$

$$\lim_{x\to-\infty} \frac{|x|\sqrt{5-\frac{2}{x^2}}}{x\left(1+\frac{3}{x}\right)}$$

$$= \lim_{x\to-\infty} - \frac{\sqrt{5-\frac{2}{x^2}}}{1+\frac{3}{x}}$$

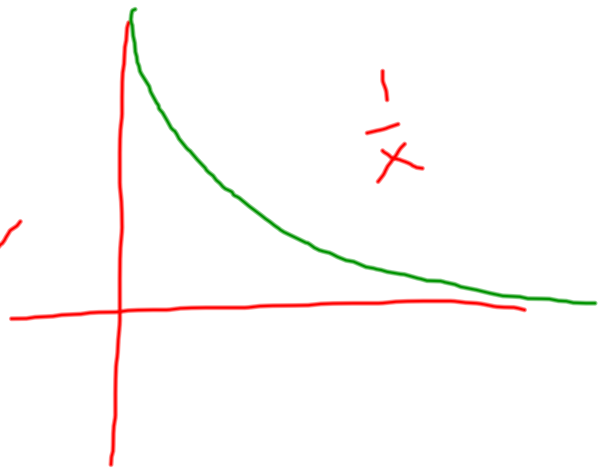
$$= -\sqrt{5}$$

$$\sqrt{x^2} = |x|$$

11) $\lim_{x\to\infty} \dfrac{3x+1}{2x-5}$

$= \lim_{x\to\infty} \dfrac{x(3+\frac{1}{x})}{x(2-\frac{5}{x})}$

$= \lim_{x\to\infty} \dfrac{3+\frac{1}{x}}{2-\frac{5}{x}}$




$= \dfrac{\lim_{x\to\infty} 3 + \lim_{x\to\infty} \frac{1}{x}}{\lim_{x\to\infty} 2 - \lim_{x\to\infty} \frac{5}{x}} = \dfrac{3+0}{2-0} = \dfrac{3}{2}$

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12) $\lim\limits_{x \to \infty} \dfrac{5x^2 - 4x}{2x^2+3}$

$$\lim_{x \to \infty} \frac{x^2\left(5 - \frac{4}{x}\right)}{x^2\left(2 + \frac{3}{x^2}\right)}$$

$$= \lim_{x \to \infty} \frac{5 - \frac{4}{x}}{2 + \frac{3}{x^2}} = \frac{5}{2}$$

19

$$\lim_{x \to -\infty} \frac{+\sqrt{5x^2-2}}{-\,x+3} \;\; \begin{matrix} \rightarrow +\infty \\ \rightarrow -\infty \end{matrix} \qquad \frac{\infty}{\infty} \text{ idea } \underline{\text{IDK}}$$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2\left(5-\frac{2}{x^2}\right)}}{x\left(1+\frac{3}{x}\right)}$$

$$\sqrt{2^2} = 2$$

$$\cancel{\sqrt{(-2)^2} = 2 = |-2|}$$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2}\sqrt{5-\frac{2}{x^2}}}{x\left(1+\frac{3}{x}\right)}$$

+ numeric

$$\lim_{x \to -\infty} -\frac{\sqrt{5-\frac{2}{x^2}}}{\left(1+\frac{3}{x}\right)}$$

$$\lim_{x \to -\infty} \frac{|x|\sqrt{5-2/x^2}}{x\left(1+\frac{3}{x}\right)}$$

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$$= -\frac{\sqrt{5}}{1} = -\sqrt{5}$$

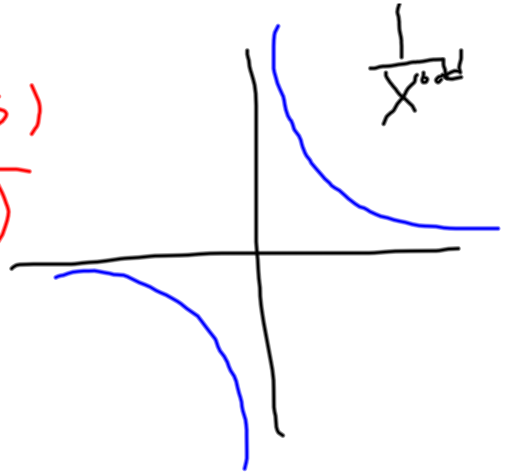
17] $\lim\limits_{x\to+\infty} \sqrt[3]{\dfrac{2+3x-5x^2}{1+8x^2}}$

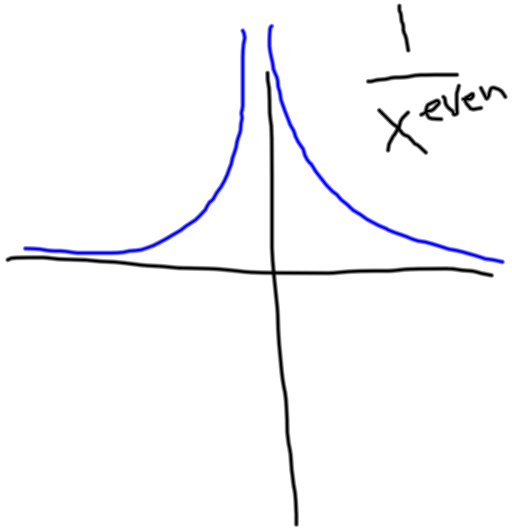
$\sqrt[3]{\dfrac{x^2\left(\frac{2}{x^2}+\frac{3}{x}-5\right)}{x^2\left(\frac{1}{x^2}+8\right)}}$

$= \sqrt[3]{\lim\limits_{x\to\infty} \dfrac{2+3x-5x^2}{1+8x^2}}$

$= \sqrt[3]{\lim\limits_{x\to\infty} \dfrac{\cancel{x^2}\left(\frac{2}{x^2}+\frac{3}{x}-5\right)}{\cancel{x^2}\left(\frac{1}{x^2}+8\right)}}$

$= \sqrt[3]{-\dfrac{5}{8}} = -\dfrac{\sqrt[3]{5}}{2}$



24 preview

$$\lim_{x \to +\infty} \frac{\sqrt{3x^4+x}}{x^2-8}$$

$$\lim_{x \to +\infty} \frac{\sqrt{x^4\left(3+\frac{1}{x^3}\right)}}{x^2\left(1-\frac{8}{x^2}\right)}$$

$$\lim_{x \to \infty} \frac{\sqrt{x^4}\sqrt{3+1/x^3}}{x^2\ \left(1-8/x^2\right)}$$

$$= \lim_{x \to \infty} \frac{x^2\ \sqrt{3+\frac{1}{x^3}}}{x^2\ \left(1-\frac{8}{x^2}\right)}$$

$$\lim_{x \to \infty} \frac{\sqrt{3+\frac{1}{x^3}}}{\left(1-\frac{8}{x^2}\right)}$$

$$= \frac{\sqrt{3}}{1} = \sqrt{3}$$

25 preview

$$\lim_{x\to+\infty} \frac{7-6x^5}{x+3} \quad \text{"}\frac{-}{}\text{"}$$

$$\lim_{x\to+\infty} \frac{x^5\left(\frac{7}{x^5}-6\right)}{x^5\left(\frac{1}{x^4}+\frac{3}{x^5}\right)}$$

$$\lim_{x\to+\infty} \frac{\frac{7}{x^5}-6}{\frac{1}{x^4}+\frac{3}{x^5}} \approx \frac{-6}{0_{rh}} = -\infty$$

Homework

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21) $\lim_{y\to-\infty} \frac{2-y}{\sqrt{7+6y^2}}$

$= \lim_{y\to-\infty} \frac{y(\frac{2}{y}-1)}{\sqrt{y^2(\frac{7}{y^2}+6)}}$

$= \lim_{y\to-\infty} \frac{y(\frac{2}{y}-1)}{|y|\sqrt{\frac{7}{y^2}+6}}$

$= \lim_{y\to-\infty} - \frac{\frac{2}{y}-1}{\sqrt{\frac{7}{y^2}+6}} = -\frac{-1}{\sqrt{6}} = \frac{1}{\sqrt{6}}$

23) $\lim_{x\to-\infty} \frac{\sqrt{3x^4+x}}{x^2-8}$

$\lim_{x\to-\infty} \frac{\sqrt{x^4}\sqrt{3+\frac{1}{x^3}}}{x^2(1-\frac{8}{x^2})}$

$\lim_{x\to-\infty} \frac{\cancel{x^2}\sqrt{3+\frac{1}{x^3}}}{\cancel{x^2}(1-\frac{8}{x^2})}$

$\lim_{x\to-\infty} \frac{\sqrt{3+1/x^3}}{1-8/x^2}$

$= \sqrt{3}$

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25) $\lim_{x\to\infty} \dfrac{7-6x^5}{x+3}$

$$\lim_{x\to\infty} \frac{x^5\left(\frac{7}{x^5}-6\right)}{x\left(1+\frac{3}{x}\right)}$$

$$=\lim_{x\to\infty} \frac{x^4\left(\frac{7}{x^5}-6\right)}{1+\frac{3}{x}}$$

$$=-\infty$$

27) $\lim_{x\to\infty} \dfrac{6-x^3}{7x^3+3} = -\dfrac{1}{7}$

$$\lim_{x\to\infty} \frac{x^3\left(\frac{6}{x^3}-1\right)}{x^3\left(7+\frac{3}{x^3}\right)}$$

$$\lim_{x\to\infty} \frac{\frac{6}{x^3}-1}{7+\frac{3}{x^3}} = -\frac{1}{7}$$

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2a) $f(x) = \begin{cases} 2x^2+5, & x<0 \\ \dfrac{3-5x^3}{1+4x+x^3}, & x \geq 0 \end{cases}$

a) $\lim\limits_{x \to -\infty} f(x)$

$\lim\limits_{x \to -\infty} 2x^2+5$

$= \infty$

b) $\lim\limits_{x \to \infty} f(x)$

$\lim\limits_{x \to \infty} \dfrac{3-5x^3}{1+4x+x^3}$

$\lim\limits_{x \to \infty} \dfrac{\cancel{x^3}\left(\frac{3}{x^3}-5\right)}{\cancel{x^3}\left(\frac{1}{x^3}+\frac{4}{x^2}+1\right)} = -5$

30) $g(t) = \begin{cases} \dfrac{2+3t}{5t^2+6}, & t < 10^6 \\ \dfrac{\sqrt{36t^2-100}}{5-t}, & t > 10^6 \end{cases}$

a) $\lim\limits_{t\to-\infty} g(t)$

$$\lim_{t\to-\infty} \frac{2+3t}{5t^2+6}$$

$$\lim_{t\to-\infty} \frac{\frac{2}{t}+3}{t\left(5+\frac{6}{t^2}\right)} = 0$$

b) $\lim\limits_{t\to\infty} g(t)$

$$\lim_{t\to\infty} \frac{\sqrt{36t^2-100}}{5-t}$$

$$= \lim_{t\to\infty} \frac{|t|\sqrt{36-\frac{100}{t^2}}}{t\left(\frac{5}{t}-1\right)} = \frac{\sqrt{36}}{-1} = -6$$

31) $\lim_{x\to+\infty}\left(\sqrt{x^2+3}-x\right)$ $\infty-\infty$ indeterminate form

$\lim_{x\to+\infty}\frac{\sqrt{x^2+3}-x}{1}\cdot\frac{\left(\sqrt{x^2+3}+x\right)}{\left(\sqrt{x^2+3}+x\right)}$

$=\lim_{x\to+\infty}\frac{(x^2+3)-x^2}{\sqrt{x^2+3}+x}=$

$\lim_{x\to\infty}\frac{3}{\sqrt{x^2+3}+x}$

$=0$

(2.2 39) $\lim_{x\to0}\frac{\sqrt{x+4}-2}{x}=\frac{0}{0}$ IDK

$\lim_{x\to0}\frac{\left(\sqrt{x+4}-2\right)\left(\sqrt{x+4}+2\right)}{x\left(\sqrt{x+4}+2\right)}$

$=\lim_{x\to0}\frac{(x+4)-4}{x\left(\sqrt{x+4}+2\right)}$

$=\lim_{x\to0}\frac{x}{x\left(\sqrt{x+4}+2\right)}$

$=\lim_{x\to0}\frac{1}{\sqrt{x+4}+2}=\frac{1}{4}$

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30) $g(t) = \begin{cases} \dfrac{2+3t}{5t^2+6}, & t < 10^6 \\ \dfrac{\sqrt{36t^2-100}}{5-t}, & t > 10^6 \end{cases}$

a) $\lim\limits_{t\to-\infty} g(t)$

$$\lim_{t\to-\infty} \frac{2+3t}{5t^2+6}$$

$$\lim_{t\to-\infty} \frac{t\left(\frac{2}{t}+3\right)}{t^2\left(5+\frac{6}{t^2}\right)}$$

$$= \lim_{t\to-\infty} \frac{\left(\frac{2}{t}+3\right)}{t\left(5+\frac{6}{t^2}\right)} = 0$$

b) $\lim\limits_{t\to\infty} g(t)$

$$\lim_{t\to\infty} \frac{\sqrt{36t^2-100}}{5-t}$$

$$\lim_{t\to\infty} \frac{\sqrt{t^2}\sqrt{36-\frac{100}{t^2}}}{t\left(\frac{5}{t}-1\right)}$$

$$\lim_{t\to\infty} \frac{|t|\sqrt{36-\frac{100}{t^2}}}{t\left(\frac{5}{t}-1\right)} = \lim_{t\to\infty} \frac{\sqrt{36-\frac{100}{t^2}}}{\frac{5}{t}-1}$$

$$= \frac{\sqrt{36}}{-1} = -6$$

31) $\lim_{x\to\infty} \left(\sqrt{x^2+3} - x\right)$ $\quad \infty - \infty$

"indeterminate form"
= IDK

$$\lim_{x\to\infty} \frac{\left(\sqrt{x^2+3} - x\right)}{1} \cdot \frac{\left(\sqrt{x^2+3} + x\right)}{\left(\sqrt{x^2+3} + x\right)}$$

$$= \lim_{x\to\infty} \frac{(x^2+3) - x^2}{\sqrt{x^2+3} + x}$$

$$= \lim_{x\to\infty} \frac{3}{\sqrt{x^2+3} + x}$$

$$= 0$$

$$\lim_{x\to\infty} x\left(\sqrt{1+\frac{3}{x^2}} - 1\right)$$

$\infty \cdot 0$
indeterminate form

2.2 39*) $\lim_{x\to 0} \frac{\sqrt{x+4} - 2}{x}$

$$\lim_{x\to 0} \frac{\left(\sqrt{x+4} - 2\right)\left(\sqrt{x+4} + 2\right)}{x\left(\sqrt{x+4} + 2\right)}$$

$$\lim_{x\to 0} \frac{(x+4) - 4}{x\left(\sqrt{x+4} + 2\right)} = \lim_{x\to 0} \frac{x}{x\left(\sqrt{x+4} + 2\right)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x+4} + 2} = \frac{1}{4}$$

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39H $\lim_{x\to 0} \frac{\left(\sqrt{x^2+4} - 2\right)}{x} \cdot \frac{\left(\sqrt{x^2+4} + 2\right)}{\left(\sqrt{x^2+4} + 2\right)}$

$$= \lim_{x\to 0} \frac{(x^2+4) - 4}{x\left(\sqrt{x^2+4} + 2\right)} = \lim_{x\to 0} \frac{x^2}{x\left(\sqrt{x^2+4} + 2\right)}$$

$$= \lim_{x\to 0} \frac{x}{\sqrt{x^2+4} + 2} = \frac{0}{4} = 0$$

$$= \lim_{x \to 0} \frac{x}{\sqrt{x^2+4} + 2}$$

$$\lim_{x \to 0} \frac{x(1)}{\sqrt{x^2}\left(\sqrt{1+\frac{4}{x^2}}\right) + 2}$$

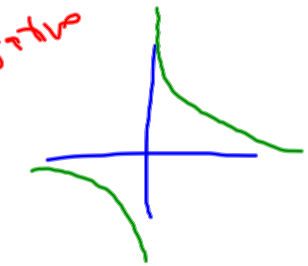
$$\lim_{x \to 0^+} \frac{x(1)}{x\left[\sqrt{1+\frac{4}{x^2}} + \frac{2}{x}\right]}$$

$$\lim_{x \to 0^+} \frac{1}{\sqrt{1+\underbrace{\frac{4}{x^2}}_{\infty}} + \underbrace{\frac{2}{x}}_{\infty}} = 0$$

$$\frac{1}{\infty}$$

$\sqrt{x^2} = -x$ when $x$ is negative

$= |x|$



$$\lim_{x \to 0^-} \frac{x(1)}{|x|\sqrt{1+\frac{4}{x^2}} + 2}$$

$$\lim_{x \to 0^-} \frac{x(1)}{-x\sqrt{1+\frac{4}{x^2}} + 2}$$

$$\lim_{x \to 0^-} \frac{x(1)}{x\left(-\sqrt{1+\frac{4}{x^2}} + \frac{2}{x}\right)}$$

$$\lim_{x \to 0^-} \frac{1}{-\sqrt{1+\frac{4}{x^2}} + \frac{2}{x}} = 0$$